Mental Math

Fact Learning
Mental Computation
Estimation

Grade 2
Teacher's Guide

Prince Edward Island
CANADA
Department of Education
English Programs

Mental Math

Fact Learning
Mental Computation
Estimation

Grade 2
Teacher's Guide

Department of Education

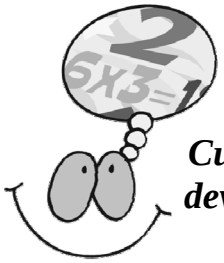
English Programs
PO Box 2000
Charlottetown, PE C1A 7N8
Tel: (902) 368 4600
Fax: (902) 368 4622
www.gov.pe.ca/edu

2008

Table of Contents

Mental Math in the Elementary Mathematics Curriculum

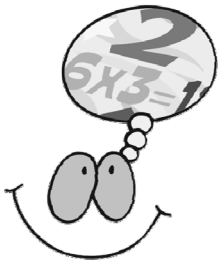
Mental math in this guide refers to fact learning, mental computation, and computational estimation. The Atlantic Canada Mathematics Curriculum supports the acquisition of these skills through the development of thinking strategies across grade levels.



Mental math refers to fact learning, mental computation, and computational estimation. The Atlantic Canada Mathematics Curriculum supports the acquisition of these skills through the development of thinking strategies across grade levels.

Pre-Operational Skills

Many children begin school with a limited understanding of number and number relationships. Counting skills, which are essential for ordering and comparing numbers, are an important component in the development of number sense. Counting on, counting back, concepts of more and less, and the ability to recognize patterned sets, all mark advances in children's development of number ideas.

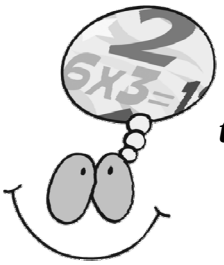


Basic facts are mathematical operations for which some students may not be conceptually prepared.

Basic facts are mathematical operations for which some students may not be conceptually prepared. As a minimum, the following skills should be in place before children are expected to acquire basic facts.

- Students can immediately name the number that comes after a given number from 0-9, or before a given number from 2-10.
- When shown a familiar arrangement of dots $\leq$ 10 on ten frames, dice, or dot cards, students can quickly identify the number without counting.

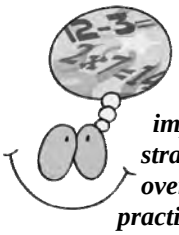
- For numbers $\leq$ 10 students can quickly name the number that is one-more, one-less; two-more, two-less. (the concept of less tends to be more problematic for children and is related to strategies for the subtraction facts)



Mental mathematics must be a consistent part of instruction in computation from primary through the elementary and middle grades.

Curriculum Outcomes	Thinking Strategies
Grade 1 **B7-** use mental strategies to find sums to 18 and differences from 18 or less **B8-** memorize simple addition and/or subtraction facts from among those for which the total is 10 or less **C5-** use number patterns to help solve addition and subtraction sentences	**P. 28** • Doubles Facts for addition and subtraction facts **P. 36** • Using patterns to learn the facts • Commutative property (3+2 = 2+3)
Grade 2 **B5-** develop and apply strategies to learn addition and subtraction facts **B11-** estimate the sum or difference of two 2-digit numbers  ***Fact learning is a mental exercise with an oral and/or visual prompt; the focus is oral, rather than paper-and pencil; drills should be short with immediate feedback over an extended period of time.***	**P. 22** • Doubles plus 1 • Make 10 (“bridging to 10”) • Two-apart facts; double in-between • Subtraction as “think addition” • Compensation • Balancing for a constant difference **P. 30** (Estimation) • Rounding both numbers to the nearest 10 • Round one number up and one number down • Front-end estimation
Grade 3 **B11/12-** mentally add and subtract two-digit and one-digit numbers, and rounded numbers. **B9-** continue to estimate in addition and subtraction situations **B10-** begin to estimate in multiplication and division situations **C3 -** use and recognize the patterns in a multiplication table	**P. 34** • Make 10 • Compatible numbers (“partner” numbers) • Front-end addition • Back up through ten (“counting on”) • Compensation • Balancing for a constant difference **P. 28** • Commutative property for multiplication (3x2 = 2x3) • Division as “think multiplication” • Helping facts

Curriculum Outcomes	Thinking Strategies
Grade 4 **B9 -** demonstrate a knowledge of the multiplication facts to 9 x 9 **B14 -** estimate the product or quotient of 2- or 3-digit numbers and single digit numbers **B15 -** mentally solve appropriate addition and subtraction computations **B16 -** mentally multiply 2-digit numbers by 10 or 100 **C2 -** apply the pattern identified when multiplying by increasing powers of 10	**P. 32** • Doubles • Clock-facts for 5's • Patterns for 9's • Helping facts **P. 36** (Estimation) • Rounding • Front-end • Clustering of Compatibles **P. 38** • Compatibles for division **P. 40** • Front-end addition • Compensation • Up through 100 (counting on) • Back down through 100 (counting back) • Compatible numbers • Place-value-change strategy for mentally multiplying by 10, 100

Curriculum Outcomes	Thinking Strategies
Grade 5 **B10-** estimate sums and differences involving decimals to thousandths **B11-** estimate products and quotients of two whole numbers **B12-** estimate products and quotients of decimal numbers by single-digit whole numbers **B15-** multiply whole numbers by 0.1, 0.01, and 0.001 mentally **C2-** recognize and explain the pattern in dividing by 10, 100, 1000 and in multiplying by 0.1, 0.01 and 0.001 **B13-** perform appropriate mental multiplications with facility  ***By grade 5, students should possess a variety of strategies to compute mentally. It is important to recognize that these strategies develop and improve over the years with regular practice.***	**P. 40 to 41** (Estimation) • Rounding one up, one down • Looking for compatibles that make approximately 10, 100, 1000 • Front-end **P. 44** • Place-value-change strategy for mentally multiplying by 10, 100, 1000 • "Halve-double" strategy for multiplication • Front-end multiplication • Compensation **P. 46 to 50** • Place-value-change strategy for mentally dividing by 10, 100, 1000 • Place-value-change strategy for mentally multiplying by 0.1, 0.01, 0.001

Grade 6 **B9-** estimate products and quotients involving whole numbers only, whole numbers and decimals, and decimals only **B10-** divide numbers by 0.1, 0.01, and 0.001 mentally **C2-** use patterns to explore division by 0.1, 0.01, and 0.001 **B11-** calculate sums and differences in relevant contexts using the most appropriate method	**P. 40** (Estimation) • Rounding one up, one down for multiplication • Front-end method for multiplication and division **P. 42 and 50** • Place-value-change strategy for mentally dividing by 0.1, 0.01, 0.001 **P. 44** • Compensation in multiplication • Front-end

Students should perform mental computations with facility using strategies outlined in the Mental Math Guides.

Definitions and Connections

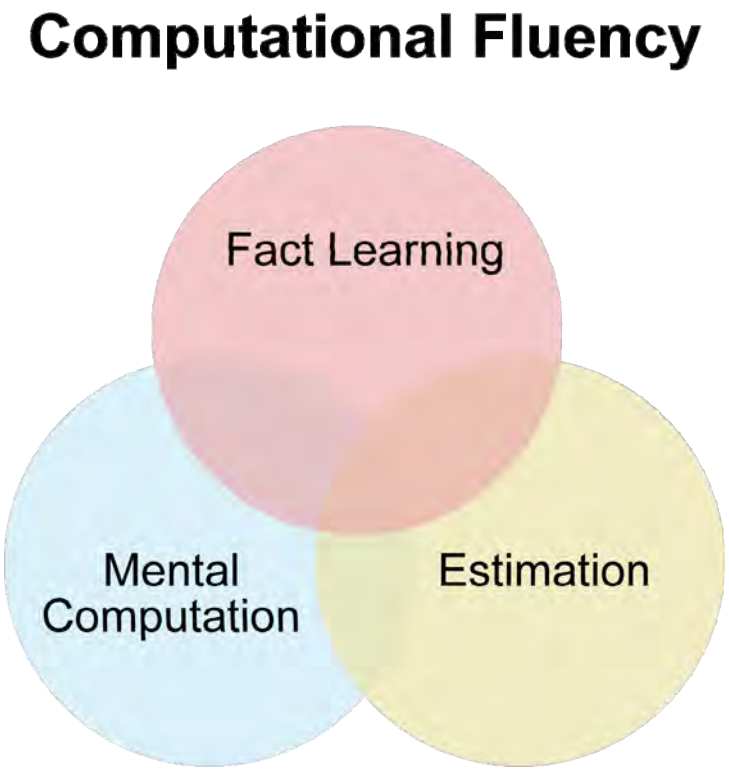
Fact learning refers to the acquisition of the 100 number facts relating to the single digits 0-9 in each of the four operations. Mastery is defined by a correct response in 3 seconds or less.

Mental computation refers to using strategies to get exact answers by doing most of the calculations in one's head. Depending on the number of steps involved, the process may be assisted by quick jottings of sub-steps to support short term memory.

Computational estimation refers to using strategies to get approximate answers by doing calculations mentally.

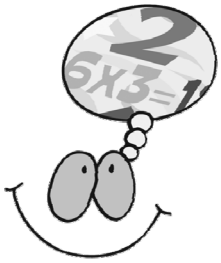
Students develop and use thinking strategies to recall answers to basic facts. These are the foundation for the development of other mental calculation strategies. When facts are automatic, students are no longer using strategies to retrieve them from memory.

Basic facts and mental calculation strategies are the foundations for estimation. Attempts at estimation are often thwarted by the lack of knowledge of the related facts and mental math strategies.



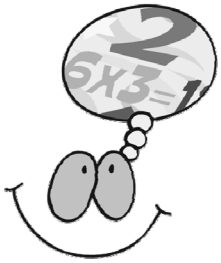
Rationale

In modern society, the development of mental computation skills needs to be a goal of any mathematical program for two important reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people still need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.



In modern society, the development of mental computation skills needs to be a goal of any mathematics program.

Besides being the foundation of the development of number and operation sense, fact learning is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these are numerical. Without a command of the basic facts, it is very difficult to detect these patterns and relationships. As well, nothing empowers students more with confidence, and a level of independence in mathematics, than a command of the number facts.



...nothing empowers students more with confidence, and a level of independence in mathematics, than a command of the number facts.

Introducing Thinking Strategies to Students

Understanding our base ten system of numeration is key to developing computational fluency. At all grades, beginning with single digit addition, the special place of the number 10 and its multiples is stressed. In addition, students are encouraged to add to make 10 first, and then add beyond the ten. Addition of ten and multiples of ten is emphasized, as well as multiplication by 10 and its multiples.

Relationships that exist between numbers and among number facts should be used to facilitate learning. The more connections that are established, and the greater the understanding, the easier it is to master facts. For example, students learn that they can get to 3 + 4 if they know 3 + 3, because 3 + 4 is one more than double 3.

When introducing and explaining a thinking strategy, include anything that will help students see its pattern, logic, and simplicity. The more senses you can involve when introducing the facts, the greater the likelihood of success for all students.

When introducing and explaining a thinking strategy, include anything that will help students see its pattern, logic, and simplicity. The more senses you can involve when introducing the facts, the greater the likelihood of success for all students. Many of the thinking strategies, supported by research and outlined in the mathematics curriculum, advocate for a variety of learning modalities. For example:

- *Visual* (images for the addition doubles)
- *Auditory* (silly sayings and rhymes) "*4 + 4, there's a spider on my door*."
- *Patterns in Number*
- *Tactile* (ten-frames, base ten blocks)
- *Helping Facts* (3 + 3 = 6, so 3 + 4 or 4 + 3 is one more. 3 + 4 = 7)

Teachers should also "think aloud" to model the mental processes used to apply the strategy and discuss situations where it is most appropriate and efficient as well as those in which it would not be appropriate at all.

In any classroom, there may be several students who have already mastered some or all of the single-digit number facts. Perhaps they have acquired them through drill and practice, or through songs and rhymes, or perhaps they "just know them". Whatever the case, once a student has mastered these facts, there is no need to learn new strategies for them. In other words, it is not necessary to teach a strategy for a fact that has been learned in a different way. On the other hand, all students can benefit from activities and discussions that help them understand how and why a particular strategy works. This kind of understanding is key to number sense development.

Practice and Reinforcement

While the words *drill* and *practice* are often used interchangeably, it is important to consider the useful distinction offered by John Van DeWalle in his book, Teaching Student-Centered Mathematics Grades K-3 (Pearson Education Inc. 2006).

In his view, *practice* refers to problem-based activities (simple story problems) where students are encouraged to develop their own solution strategies. They invent and try ideas that are meaningful to them, but they do not master these skills.

Drill, on the other hand, refers to repetitive non-problem-based activities appropriate for children who have a strategy that they understand, like, and know how to use, but are not yet fluent in applying. Drill with a particular strategy for a group of facts focuses students' attention on that strategy and helps to make it more automatic.

However, not all children will be ready for drill exercises at the same time and it is critical that it not be introduced too soon. For example, suppose a child does not know the fact 9+5, and has no way to deal with it other than to employ inefficient methods such as counting on fingers or number lines. To give this child a drill exercise which offers no new information or encourages no new connections is both a waste of time and a frustration for the child. Many children will simply not be ready to use an idea the first few days and will need lots of opportunities to make the strategy their own.

It is important to remember that drill exercises should only be provided when an efficient strategy is in place.

Once a strategy has been taught, it is important to reinforce it. The reinforcement or practice exercises should be varied in type, and focus as much on the discussion of how students obtained their answers as on the answers themselves.

The selection of appropriate exercises for the reinforcement of each strategy is critical. The numbers should be ones for which the strategy being practiced most aptly applies and, in addition to lists of number expressions, the practice items should often include applications in contexts.

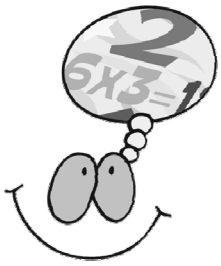
Drill exercises should be presented with both **visual** and **oral** prompts and the oral prompts that you give should expose students to a variety of linguistic descriptions for the operations. For example, 5 + 4 could be described as:

- *the sum of 5 and 4*
- *4 added to 5*
- *5 add 4*
- *5 plus 4*
- *4 more than 5*
- *5 and 4 etc*

Response Time

- *Number Facts*

In the curriculum guide, fact mastery is described as a correct response in 3 seconds or less and is an indication that the student has committed the facts to memory. This 3-second-response goal is merely a guideline for teachers and does not need to be shared with students if it will cause undue anxiety. Initially, you would allow students more time than this as they learn to apply new strategies, and reduce the time as they become more proficient.



This 3-second-response goal is merely a guideline for teachers and does not need to be shared with students if it will cause undue anxiety.

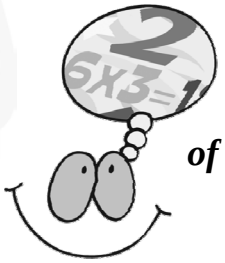
- *Mental Computation*

In grade 1, children are introduced to one mental computation strategy, *Adding 10 to a Single-Digit Number*.

Even though students in kindergarten, first and second grade *experience* numbers up to 20 and beyond on a daily basis, it should not be assumed that they understand these numbers to the same extent that they understand numbers 0-10. The set of relationships that they have developed on the smaller numbers is not easily extended to the numbers beyond 10. And yet, these numbers play a big part in many simple counting activities, in basic facts, and in much of what we do with mental computation.

Counting and grouping experiences should be developed to the point where a set of ten plays a major role in children's initial understanding of the numbers between 10 and 20. This is not a simple relationship for many children to grasp and will take considerable time to develop. However, the goal is that when they see a set of six with a set of ten, they should *come to know*, without counting, that the total is 16.

It should be remembered, however, that this is not an appropriate place to discuss place-value concepts. That is, children should not be asked to explain that the 1 in 16 represents "one ten" or that 16 is "one ten and six ones." These are confusing concepts for young children and should not be formalized in Grade 1. Even in Grade 2 the curriculum reminds teachers that place-value concepts develop slowly and should initially center around counting activities involving different-sized groups (groups of five, groups of two, etc.) Eventually, children will be counting groups of ten, but standard column headings (Tens and Ones) should not be used too soon as these can be misleading to students.



The major objective here is helping the children make that important connection between all that they know about counting by ones and the concept of grouping by tens.

Assessment

Your assessment of fact learning and mental computation should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you provide one-at-a-time within a certain time frame, you should also record any observations you make during practice sessions.

Oral responses and explanations from children, as well as individual interviews, can provide the teacher with many insights into a student's thinking and help identify groups of students that can all benefit from the same kind of instruction and practice.

Timed Tests of Basic Facts

The thinking strategy approach prescribed by our curriculum is to teach students strategies that can be applied to a group of facts with mastery being defined as a correct response in 3 seconds or less. The traditional timed test would have limited use in assessing this goal. To be sure, if you gave your class 50 number facts to be answered in 3 minutes and some students completed all, or most, of them correctly, you would expect that these students know their facts. However, if other students only completed some of these facts and got many of those correct, you wouldn't know how long they spent on each question and you wouldn't have the information you need to assess the outcome. You could use these sheets in alternative ways, however.

For example:

- Ask students to quickly answer the facts which they know right away and circle the facts they think are "hard" for them. This type of self assessment can provide teachers with valuable information about each student's level of confidence and perceived mastery.
- Ask students to circle and complete only the facts for which a specific strategy would be useful. For example, circle and complete all the "double facts".

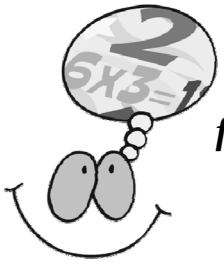
Parents and Guardians:
Partners in Developing Mental Math Skills

Parents and guardians are valuable partners in reinforcing the strategies you are developing in school. You should help parents understand the importance of these strategies in the overall development of their children's mathematical thinking, and encourage them to have their children do mental computation in natural situations at home and out in the community.

You should also help parents understand that the methods and techniques that helped them learn basic facts as students may also work for their own children and are still valuable strategies to introduce. We can never be sure which ideas will make the most sense to children, but we can always be certain that they will adopt the strategies that work best for them.

Our goal, for teachers and parents alike, is to help students broaden their repertoire of thinking strategies and become more flexible thinkers; it is not to prescribe what they must use.

Through various forms of communication, you should keep parents abreast of the strategies you are teaching and the types of mental computations they should expect their children to be able to do.



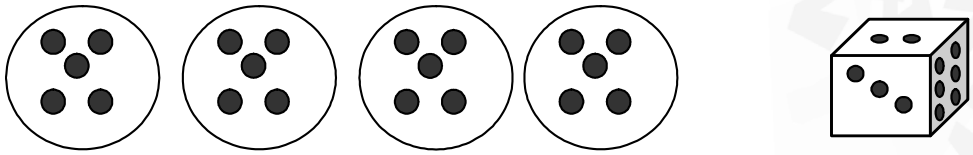
Our goal, for teachers and parents alike, is to help students broaden their repertoire of thinking strategies and become more flexible thinkers; it is not to prescribe what they must use.

Pre-Operational Skills

A. Pre-operational Skills

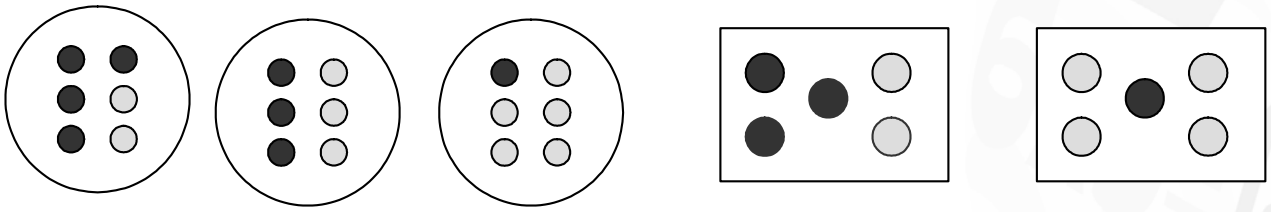
- **Patterned Set Recognition for Numbers 1-6**

  Students are able to recognize common configuration sets of numbers such as the dots on a standard die, dominoes, ten frames, and dot cards. Set recognition can be reinforced through flash math activities where students are presented with a number configuration for a few seconds, and are asked to identify the number that it represents.



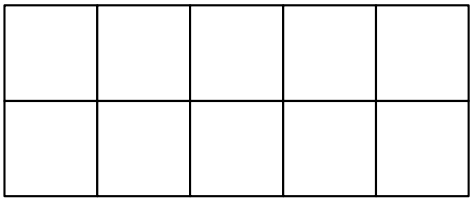
- **Part-Part-Whole Relationships**

  This relationship refers to the recognition of two parts in a whole and an understanding that numbers can be *decomposed* into parts. When shown dot patterns made up of two colours, the child might be asked, "*How many dots did you see*? *How many were red*? *How many were blue*?"



- **Ten-Frame Visualization for Numbers 0-10**

  The work students do with ten frames should eventually lead to a mental math stage where they can visualize the standard ten-frame representation of numbers and answer questions from their visual memories.



For example, you might ask students to visualize the number 8, and ask,
How many dots are in the first row?
How many are in the second row?
How many more dots are needed to make 10?
What number would you have if you added one more dot?
What number would you have if you removed 3 dots?

This activity can then be extended to identify the number sentences associated with the ten-frame representations.

For example, for the number 6 on a ten frame, students could identify these number sentences:

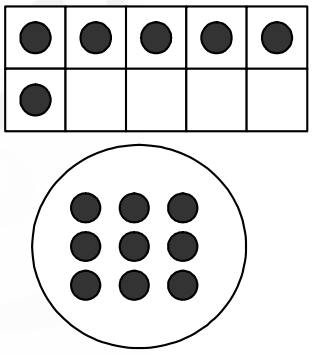
5 + 1 = 6	6 + 4 = 10
1 + 5 = 6	10 – 4 = 6
6 – 1 = 5	10 – 6 = 4
6 – 5 = 1	6 – 6 = 0

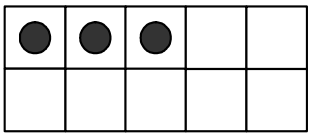
B. Other Number Relationships

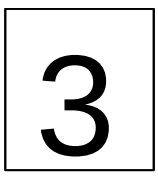
- **One More/One Less and Two More/Two Less**

Work in developing these relationships will be a major focus for the grade 1 teacher throughout the year and should eventually lead to a mental math stage where students are presented with a number and asked for the number up to 20 that is *one more, one less, two more*, or *two less* than the number.

Materials such as dominoes, dice, dot plates, playing cards, numeral cards and ten-frames can all be used to help reinforce these number relationships.







Depending on which relationship you want to reinforce, children can be asked the following kinds of questions:

- *Which number is 1 more than this*?
- *Which number is 2 more than this number*?
- *Which number is one less than this one*?
- *Which number is two less than this*?

- **Next Number and Counting On and Back**

  The ability to immediately state the number that comes after any given number from 0 – 9 is a necessary skill for learning the "plus-1 facts". As well, children's counting experiences in school should lead to a mental math stage where they can, without concrete materials or number lines, count on and back from a given number 0 -10 and skip count by 2s to 20 and by 5s and 10s to 100 starting at zero.

Fact Learning

C. Fact Learning – Addition

- **Reviewing Addition Facts and Fact Learning Strategies**

In grade 1, students are to *know* simple addition facts to 10 and be able to use mental strategies for some facts to 18. The addition facts are grouped and taught in logical rather than numerical order starting with the "doubles". A *counting-on* strategy can be used for some facts but the ten-frame should also be used extensively to help students visualize the combinations that make 10.

At the beginning of grade two, it is important to review the thinking strategies for addition facts with sums to 10 and the related subtraction facts. Students are expected to be able to recall facts with sums to 10 with a three-second response by mid-year and to recall facts to 18 with a three-second response by the end of grade two.

Addition Facts With Sums to 10

Doubles 1+1 2+2 3+3 4+4 5+5	Plus 2 Facts 3+2 2+3 4+2 2+4 5+2 2+5 6+2 2+6 7+2 2+7 8+2 2+8
Plus 1Facts 2+1 1+2 3+1 1+3 4+1 1+4 5+1 1+5 6+1 1+6 7+1 1+7 8+1 1+8 9+1 1+9	Plus 3 Facts 4+3 3+4 5+3 3+5 6+3 3+6 7+3 3+7

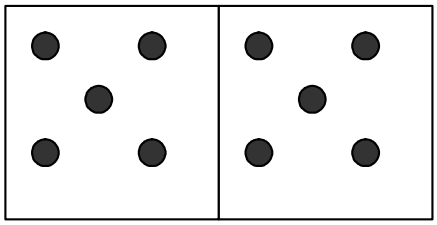
- **Thinking Strategies for Addition Fact Learning in Grade 1**

Doubles

There are only ten *doubles* from 0 + 0 to 9 + 9 and most students learn them quickly. The *doubles posters*, which have been specially created for classroom use, provide a visual context for these facts. These same posters will also be found in classrooms at the grade 3 and 4 level to teach multiplication facts that have a factor of 2. For example, the image of the 18 wheeler for the addition double 9 + 9 will be recalled when students are learning the 2-times table in multiplication; 2x9 and 9x2 is the same as"double 9".

Dot pictures (similar to dominoes, but based on the more familiar dot patterns found on number cubes) give students another way to visualize the combinations up to *double* 6.

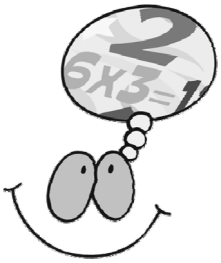
Double 5



Plus 1 Facts

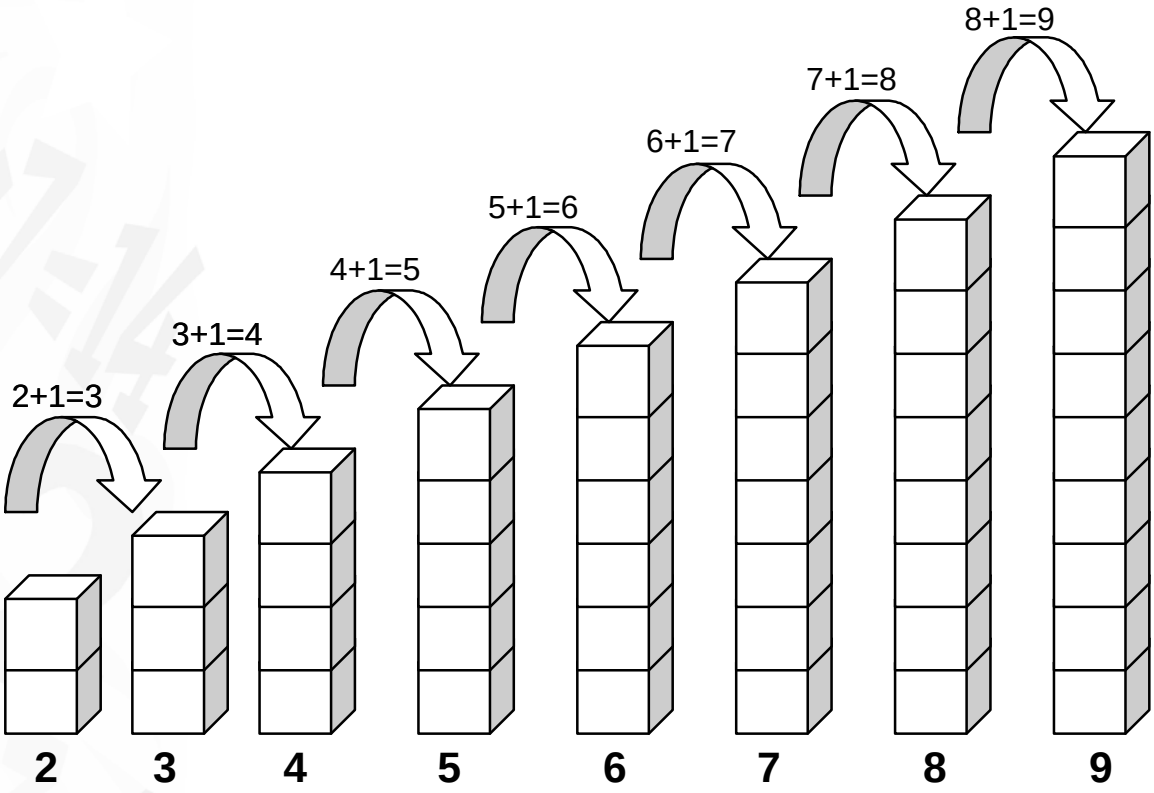
These facts are the “next number” facts. Students must be at the conceptual stage whereby they are able to say the next number after any number from 1-9 without hesitation. For any fact involving + 1, direct students to ask for the next number. For example: 7 + 1 or 1 + 7 is asking for the number after 7. Number charts and number lines help students *visualize* the +1addition facts using this strategy.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20



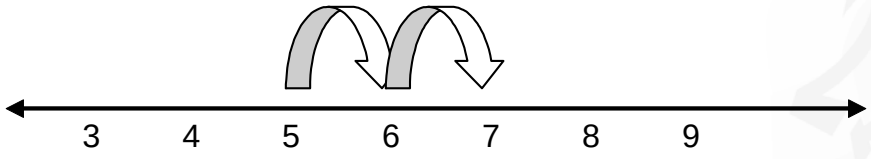
A strategy provides a mental path from the fact to the answer. Soon the fact and answer are “connected” as the strategy becomes almost unconscious.

The plus 1 facts can also be modelled using linking cubes. Have students build towers for the numbers 2 to 9. If they add one linking cube to any of these towers, they can easily see that they get the next tower. This would also be true if each of these towers were added to one cube (1+3, 1+4, 1+5, etc.)



Plus 2, Plus 3 Facts

For any number involving + 2 or + 3, direct students to think of *skip counting* by 2s or to *count on* from the larger number. An addition table and a number line can be used to help students visualize skip counting. However, students should also understand that counting on is an *inefficient* strategy for most number facts.

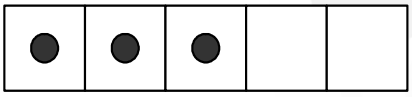


5 + 2 = 7

Using Five-Frames and Ten-Frames

Any fact which has a sum up to and including 10 can be visualized on a ten-frame. It is a good idea to start with a five-frame (half of a ten frame) to provide students with practice in visualizing facts with totals up to and including 5. The following hands-on activities should be used early in the school year with all students and as needed with individual children. Your goal is to get to the *visualization* stage where you can show students an empty frame and ask them to "see" a particular number in their minds. They then tell you how many more it would take to make 10.

- **Model Numbers on a Five-Frame**

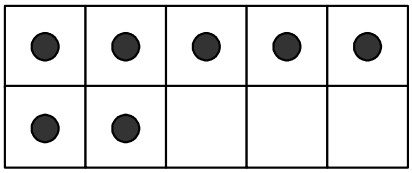


Each student works with a five-frame and counters to model numbers from 0-5. This basic activity reinforces counting and helps children see 5 as a "benchmark" number. For example, 3 is modeled in the five-frame above. Students can see that this number is made up of 1+1+1, that 2 more counters would make 5, that 5 – 2 = 3 and that 5 – 3 = 2.

- **Visualize Combinations on a Five Frame**

In this activity, students visualize the first number on a blank five-frame and say the second number that goes with it to make 5. For example, the teacher holds up a numeral card and says, “Three”.
Students “see” 3 and respond with the number of empty cells left, “*Three and **two** make* 5”.

- **Random Numbers on a Ten-Frame**



After a couple of weeks working with five-frames, introduce the ten-frame and the “rule” for showing numbers on them: *Always fill the top row first, starting on the left, the same way you read. When the top row is full, counters can be placed in the bottom row, also from the left*. The teacher calls out numbers (or holds up numeral cards, or both) and students use counters to represent each number.

- **Ten-Frame Flash Cards**

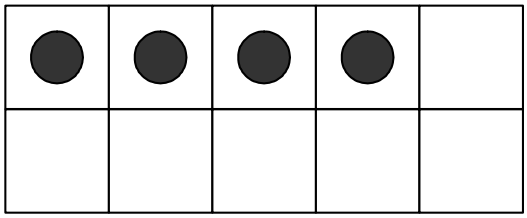
Prepare a set of 20 ten frames showing 0 and 10, and two each of the numbers 1-9. Show each card briefly and have students identify the number of dots without counting. Encourage students to explain how they saw the number. For example, how did they know it was 6 without counting each one? Discussions such as this focus on the number relationships inherent in the ten-frames and help students develop an understanding of 5 and 10 as *benchmarks* in our number system.

- **Ten-Frame Facts**

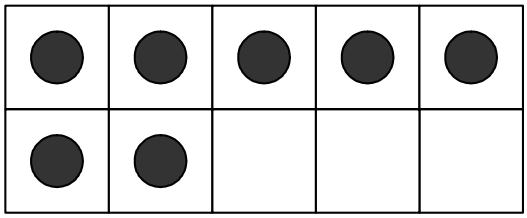
The ten-frame helps children learn the combinations that make 10. It immediately models all the combinations from 5 + 1 to 5 + 5 and their turnarounds. Even 5 +6, 5 +7, and 5 + 8 are quickly thought of as “two fives (10) and some more” when depicted with this powerful model.

Once students have had considerable experience with modelling and identifying numbers in ten-frames, it is important to focus on

combinations that make 10. Hold up a ten-frame, such as 4, and ask students to say the combination of dots and spaces that make 10, in this case, "*4 and 6 make 10*". Gradually encourage the use of the terms *plus* and *equals* to create number sentences. Repeat for other combinations.



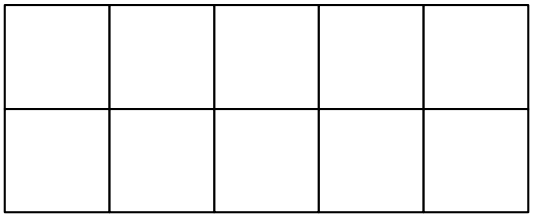
4 plus 6 equals 10.
6 plus 4 is 10.



7 plus 3 equals 10.
3 plus 7 equals 10

- **Empty Ten-Frame Facts**

  Work with ten-frames should eventually lead to the visualization stage where students can look at an empty ten-frame, "see" the number you call out, and then say the other number that combines with it to make 10. For this activity, it's a good idea to prepare a large empty ten-frame out of chart paper so that it can be displayed in a prominent location in the classroom. For example, the teacher calls out "*Seven*", and students respond with, "*Seven plus three equals ten*". Encourage children to refer to the empty ten-frame whenever they are working with numbers.



Eight! *Eight plus two equals 10!*

Three! *Three plus seven equals 10!*

New Thinking Strategies for Addition Fact Learning Introduced in Grade 2

- **Near–Doubles (1–Apart Facts)**
  The near-doubles are also called the "doubles plus one" facts and include all combinations where one addend is one more than the other. The strategy is to double the smaller number and add one. For example, 6 + 7 is the same as “double 6 plus 1 more.”
  Help students apply this strategy by modelling the following oral response: Say, "*6 + 6 is double six (12) and 1 more is 11*."

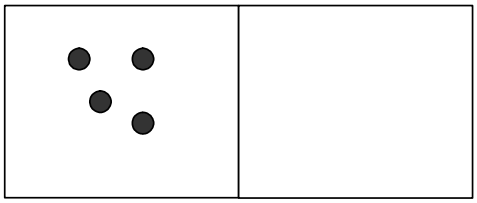
- **2–Apart Facts (Double In-Between Facts)**
  There are two effective strategies for solving addition facts whose addends differ by 2. Either double the number in between, or double the smaller number and add 2. For example, in 7 + 9, the number in between 7 and 9 is 8, and double 8 is 16. You could also use the strategy of “double 7 and add 2”.

  A significant challenge for students will be to recognize that the addends differ by two in the first place. Strategy selection activities will encourage students to look for number relationships for which a particular strategy works. For example, when shown a random collection of number facts, students might be asked to draw a circle around all the “near doubles” and a line under all the “2-apart facts”.

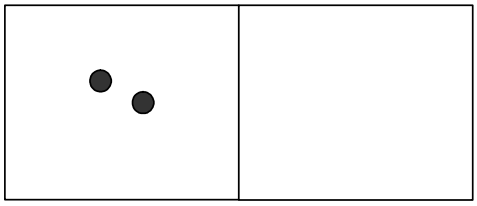
- **Plus or Minus 0 (No Change Facts)**
  Nineteen facts have zero as one of the addends. Though such number facts are generally easy to learn, some students over-generalize the idea that “plus makes numbers bigger” or “minus makes numbers smaller”. Instead of making arbitrary “rules” about adding or subtracting zero, help students build understanding by having them model simple story problems using counters and a two-part mat.

  Examples

  Mark found 4 golf balls on Saturday. (student puts 4 counters on one side of the mat).
  He didn’t find any balls on Sunday. How many balls did Mark find altogether? (student is unable to put any counters on the other part of the mat)
  Just 4. There is no change!



  Joseph bought 2 fruit roll-ups on Monday.
  On Tuesday, he bought 0. How many fruit roll-ups did Joseph buy altogether?
  Just 2. There is no change!

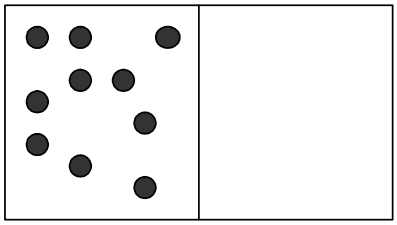


• Relationships for Numbers 10 – 20

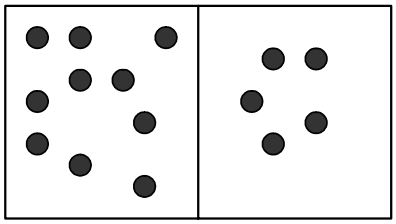
A set of 10 should play a major role in student's initial understanding of numbers between 10 and 20 and it is in grade 1 that this relationship is first explored. Although students may not yet have a complete development of place-value concepts, when they see a set of 10 and a set of 5, they should come to know that the total is 15, without counting!

Developing the Concept With 2-Part Mats

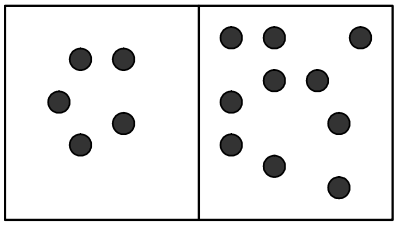
Have students count out 10 counters onto one side of a two-part mat.



Then have them put 5 counters on the other side and count all the counters by ones, "*One, two, three, four ...fifteen. Ten and five is 15*"

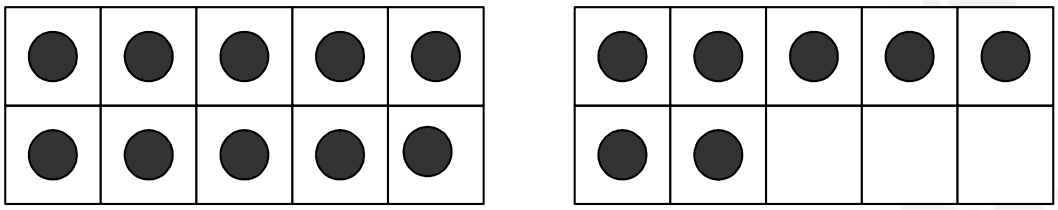


Turn the mats around. "*Five and ten is 15*". Repeat for other numbers in random order, but without changing the ten side of the mat.

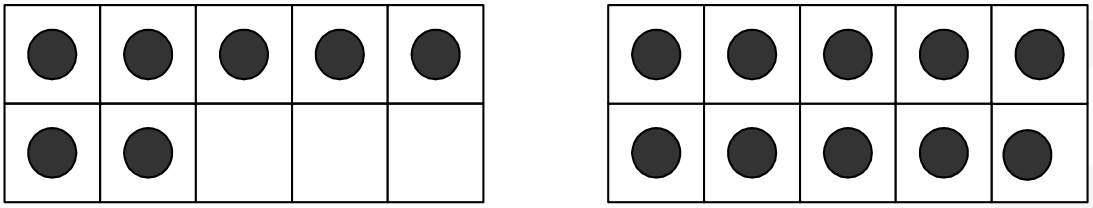


Developing the Concept With Ten-Frames

The ten-frame is an excellent model for developing the pre-place value relationship with 10. For example, present the addition sentence 10 + 7 to students and have them model it using two ten-frames.



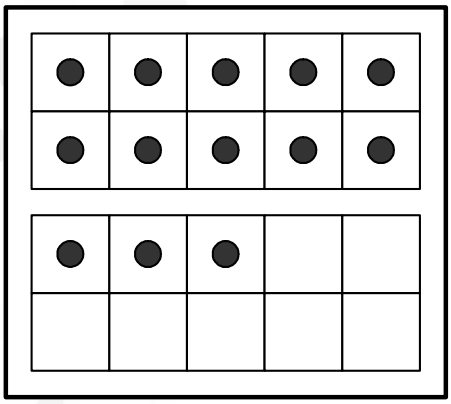
After they place 10 counters in the first ten-frame and 7 counters in the second ten-frame, ask them to name the sum represented by 10 + 7. Check if they are able to answer 17 without counting.



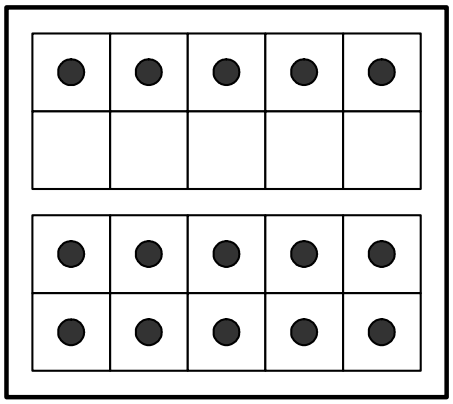
Now, have the students turn the ten-frames around and ask them to name the sum represented by these ten-frames *(7 and 10 is 17).* Continue this activity for other numbers between 10 and 20 until students no longer have to count.

Reinforcing the Concept

Practice this relationship with 10 by playing “Ten-Frame Flash” for the numbers from 10 to 20. Show the students two ten-frames for a few seconds (make sure one of the ten-frames has 10 dots). The ten-frame with the 10 dots can be the first or second one shown. Ask the students how many dots they see.



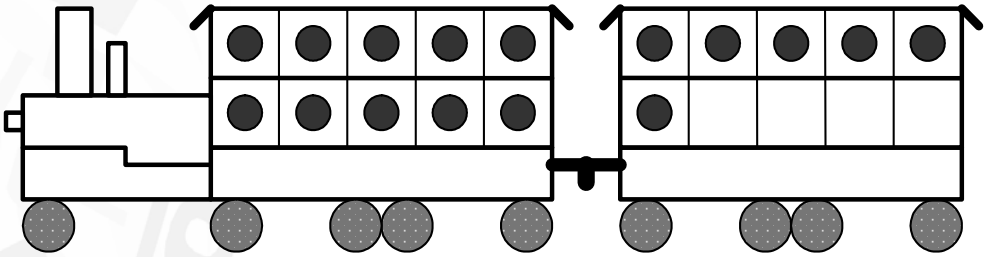
13!



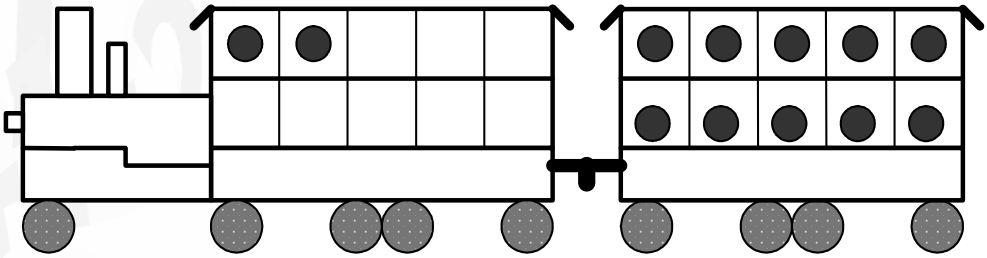
15!

Variation

You can also pretend that the ten-frames are part of a train. Draw two train cars with the ten-frames as the windows and the dots as the passengers. Place 10 passengers in the first train and 10, or fewer, passengers in the second train. Have the students name the number of passengers in the two trains.



16 people!



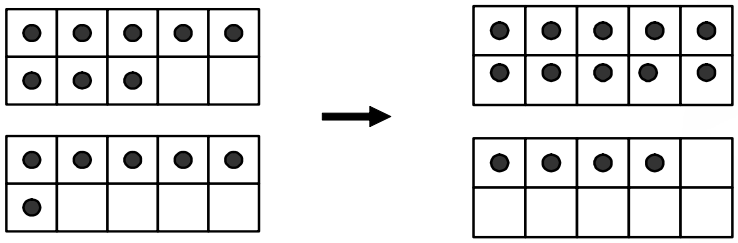
12 people!

- **Make 10 Facts**

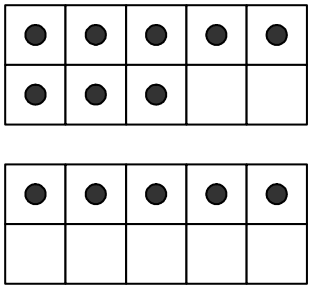
Make Ten is a thinking strategy introduced in grade 2 for addition facts which have an 8 or a 9 as one of the addends and can even be extended to facts which involve a 7.

To help develop this strategy, students use two ten frames and counters to model “Make Ten” number facts (8 + 4, 5 + 9, 6 + 8, etc.) and then rearrange the counters so that the facts read as “10 plus some more”.

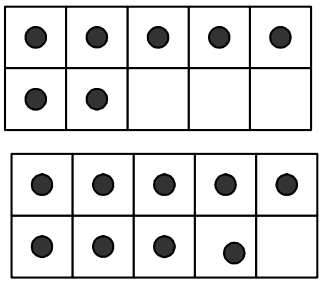
For example, students model the “make ten” fact 8 + 6 with 8 counters on one ten frame and 6 on the other. Then they move 2 counters from the 6 and give it to the 8 to make 10 + 4. Students should understand that the purpose of this strategy is to get a 10 which is easy to add.



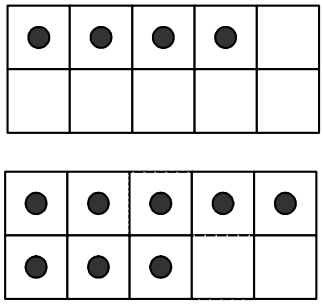
- **Make ten Flash Cards**



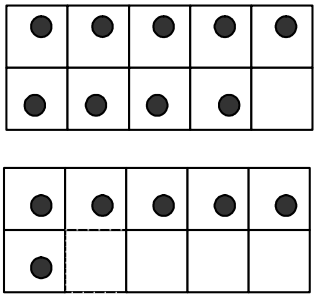
8 + 5 =



7 + 9 =



4 + 8 =



9 + 6 =

Considerable work with ten frames is required to help students understand the relationship before they are expected to perform the process mentally.

Addition Facts With Sums to 18

Doubles
1+1
2+2
3+3
4+4
5+5
6+6
7+7
8+8
9+9

Near Doubles
2+3 3+2
3+4 4+3
4+5 5+4
5+6 6+5
6+7 7+6
7+8 8+7
8+9 9+8

Plus 1 Facts
2+1 1+2
3+1 1+3
4+1 1+4
5+1 1+5
6+1 1+6
7+1 1+7
8+1 1+8
9+1 1+9

Plus 2 Facts
3+2 2+3
4+2 2+4
5+2 2+5
6+2 2+6
7+2 2+7
8+2 2+8
9+2 2+9

Plus 3
4+3 3+4
5+3 3+5
6+3 3+6
7+3 3+7
8+3 3+8
9+3 3+9

2-Apart Facts
1+3 3+1
2+4 4+2
3+5 5+3
4+6 6+4
5+7 7+5
6+8 8+6
7+9 9+7

Plus or Minus 0
Have students model simple story problems using counters and a two-part mat. For example, "*Mark found 4 golf balls on Saturday (student puts 4 counters on one side of the mat). He didn't find any balls on Sunday. How many balls did Mark find altogether?*" (*Student is unable to put any counters on the other part of the mat, so the total answer remains 4*).

Make 10 Facts
2+8 8+2
3+8 8+3
4+8 8+4
5+8 8+5
6+8 8+6
7+8 8+7
9+8 8+9
2+9 9+2
3+9 9+3
4+9 9+4
5+9 9+5
6+9 9+6
7+9 9+7
7+3 3+7
4+7 7+4
5+7 7+5
6+7 7+6
7+7

D. Fact Learning – Subtraction

- **Subtraction as "Think Addition"**

In grade 2, the expectation is that children will have *mastery* of the addition and subtraction facts to 10 by mid year and *mastery* of the addition and subtraction facts to 18 by the end of the year. As students master groups of addition facts, it is appropriate to introduce the related subtraction facts as *"think addition"* so that they can apply their knowledge in a different way. For example, if students have mastered the *Make Ten* facts, they should be presented with subtraction facts such as 15 – 8 and encouraged to think, "*8 plus what equals 15? 8 and 2 makes 10, and 5 more makes 15 so that's 7 altogether."*

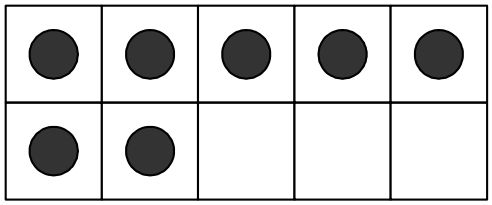
Teachers should use *think-aloud* to model this strategy.

15 - 8 =	15 - 7=
8 plus what equals 15?	*7 plus what is 15?*

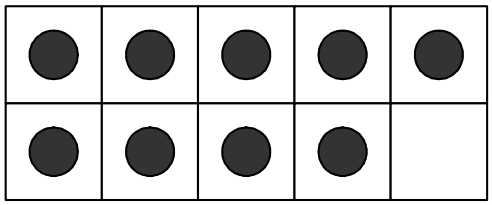
- **Ten-Frame Visualization**

Students should be able to complete many of the subtraction facts to 10, by visualizing the first number (the *minuend*) on a ten-frame and "removing" the number of dots (the *subtrahend)* to get the result (the *difference*).

Before moving to this stage, however, it is important to work with ten-frame flash cards and subtraction facts displayed vertically and horizontally. For example, hold up a ten frame with 7 dots and the subtraction fact 7 – 4 =. Students "remove" 4 dots to get the answer, "*Three*". Eventually, students will reach the stage where they are able to work with an empty ten-frame to achieve the same result.

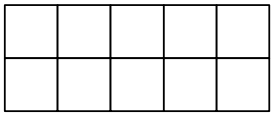


7 – 3 =
7 – 5 =
7 – 6 =
7 – 2 =



9 – 7 =
9 – 2 =
9 – 5 =
9 – 4 =

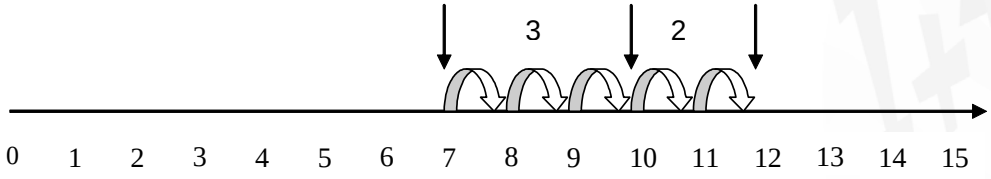
Eventually, students will reach the stage where they are able to work with an empty ten frame to achieve the same result.



- **Other Ways to Think About Subtraction**
  Besides "think addition", there are other strategies that will help students master the subtraction facts.

  - ***Up Through 10***: This strategy involves counting the difference between the two numbers by starting with the smaller number, keeping track of the *distance* to 10, and then adding this amount to the rest of the distance to the greater number. A number line or a hundreds chart can be used to develop this strategy.

Examples
For 12 – 7, think, "*Starting at 7, it's 3 to get to 10 and then 2 more to get to 12, so that's 5 altogether* "

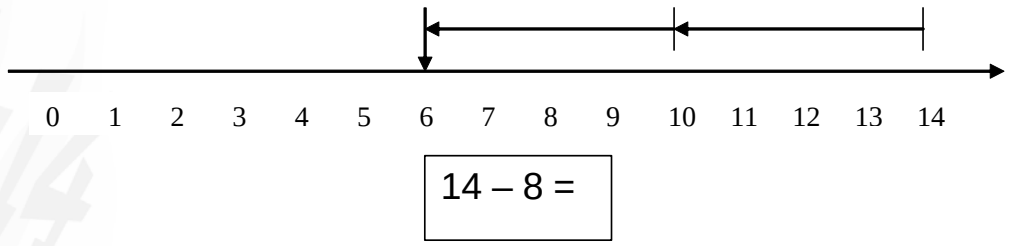


12 – 7 =

- ***Back Down Through 10***: With this strategy, you start with the larger number and subtract part of the subtrahend to get to 10, and then subtract the rest of the subtrahend.

Examples

For 14 – 8, think "*14 subtract 4 (one part of the 8) gets me to 10, and then 4 more (the rest of the 8) takes me to 6."*



For 13 – 4, think, "*13 subtract 3 is 10, and then 1 more takes me to 9*"

Subtraction facts prove to be more difficult than addition. This is especially true when children have been taught subtraction through a "count-count-count: approach; for 9 - 5, *count* out 9, *count* off 5, *count* what's left. There is little evidence that anyone who has mastered subtraction facts has found this approach helpful. In fact, children learn very few, if any, subtraction facts without first mastering the corresponding addition facts.

What may be most important is to listen to children's thinking as they attempt to answer subtraction facts that they have not yet mastered. If they are not using *think addition* or *ten-frame visualization* as a strategy, it is a good bet that they are counting - an *inefficient* method for most facts.

Subtraction Facts With Minuends to 18

Doubles		**Plus 2 Facts**		**Make Ten Facts**	
2-1	12-6	5-2	5-3	10-2	10-8
4-2	14-7	6-2	6-4	11-3	11-8
6-3	16-8	7-2	7-5	12-4	12-8
8-4	18-9	8-2	8-6	13-5	13-8
10-5		9-2	9-7	14-6	14-8
		10-2	10-8	15-7	15-8
Near Doubles				17-9	17-8
5-2	5-3	**Plus 3 Facts**		11-2	11-9
7-3	7-4	7-3	7-4	12-3	12-9
9-4	9-5	8-3	8-5	13-4	13-9
11-5	11-6	9-3	9-6	14-5	14-9
13-6	13-7	10-3	10-7	15-6	15-9
15-7	15-8	11-3	11-8	16-7	16-9
17-8	15-9	12-3	12-9	10-3	10-7
				11-4	11-7
Plus 1 Facts		**2-Apart Facts**		12-5	12-7
3-1	3-2	4-3	4-1	13-6	13-7
4-1	4-3	6-4	6-2		
5-1	5-4	8-5	8-3		
6-1	6-5	10-4	10-6		
7-1	7-6	12-5	12-7		
8-1	8-7	14-6	14-8		
9-1	9-8	16-7	16-9		
10-1	10-9				

Children learn very few, if any, subtraction facts without first mastering the corresponding addition facts.

Mental Computation

E. Mental Computation – Addition

Addition Facts Extended to 2-Digit Numbers. (New)

This strategy applies to calculations involving the addition of two numbers that are multiples of 10. Students will use their knowledge of basic facts and place value to solve these problems

- ***Doubles***

Students solve problems such as 40 + 40 by thinking *"single digit addition facts"* and then applying the appropriate place value. For example, if you know that 4 + 4 = 8, then 4 tens plus 4 tens equals 8 tens or 80.

Practice Items

60 + 60 =	20 + 20 =	30 + 30 =
50 + 50 =	70 + 70 =	90 + 90 =
80 + 80 =	10 + 10 =	

Plus 1, Plus 2, Plus 3 Facts

When presented with a number combination involving 1, 2, or 3, students are directed to ***start with the larger number*** and to *count on*. An addition table, number line or hundreds chart are useful in helping students *visualize* these relationships.

Practice Items

43 + 3 =	2 + 47 =	3 + 45 =
2 + 51 =	25 + 2 =	3 + 18 =
26 +3 =	63 + 1 =	2 + 48 =
58 + 1 =	54 + 3 =	1 + 88 =

- ***Near-Doubles (1-Apart Facts)***

Help students apply their knowledge of the "near-doubles" strategy to adding numbers which are multiples of 10. ***"Think aloud"*** as you model the process for them. For example, for 20 + 30, say, *"Twenty plus thirty is the same as double 20 and 10 more; double 20 is 40, and 10 more is 50"*

Practice Items

30 + 40 =	70 + 80 =	50 + 60 =
10 + 20 =	60 + 70 =	20 + 10 =
50 + 40 =	80 + 90 =	40 + 50 =

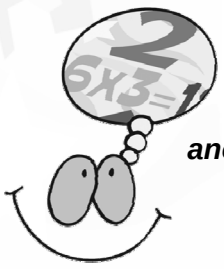
- ***2- Apart Facts***

The ***"double in-between"*** strategy works for the addition of multiples of 10 that differ by 20. For example, to add 30 + 50, think, *"Double 40 is 80, so 30 + 50 = 80"*

The *double-plus-2* strategy would also work for this type of addition problem. For example, for "30 + 50, think, *"Double 30 is 60 and 20 more is 80."* Again, it is important that the teacher ***"think aloud"*** to help students see how these strategies are being applied. Remember, however, that strategies should be introduced and developed in isolation from one another to minimize potential confusion or misunderstanding. Students will eventually select the strategy that works best for them.

Practice Items

40 + 60 =	60 + 40 =	60 + 80 =
70 + 90 =	50 + 70 =	70 + 50 =
50 + 30 =	90 + 70 =	30 + 50 =



Again, it is important that teachers "think aloud" to help students see how these strategies are being applied. Remember, however, that strategies should be introduced and developed in isolation from one another to minimize potential confusion or misunderstanding.

- ***Make 10***

This strategy is effective for addition involving 2-digit numbers which have a 7, 8, or 9 in the ones place . For example, to add 28 + 8, think, *"28 and 2 (from the 8) is 30, and 30 + 6 (the rest of the 8) is 36"*

Practice Items

2 + 18 =	8 + 19 =	47 + 8 =
19 + 8 =	17 + 6 =	27 + 6 =
17 + 5 =	18 + 8 =	39 + 8 =
4 +18 =	19 + 4 =	18 + 9 =
19 + 6 =	5 + 18 =	68 + 7 =
6 +18 =	27 + 6 =	87 + 9 =
19 + 5 =	39 + 5 =	57 + 5 =

Add your own practice items

- **Front-End Addition (New)**

This strategy is a good beginning strategy for addition (or subtraction). It involves adding the highest place values in each number first, and then adding the sums of the next place value(s).

Start by modelling the addition of two 2-digit numbers using base ten blocks. For 24 + 35, you would use 2 rods and 4 unit cubes for 24, and 3 rods, 5 unit cubes for 35. Point out that to add 24 and 35 we can combine the tens first and then the ones and rename the sum (24 + 35 = 50 + 9 = 59). Students should also be given the opportunity to model addition in this manner.

Practice Items

73 + 13 =	72 + 26 =
26 + 12 =	63 + 33 =
32 + 65 =	74 + 19 =
25 + 63 =	32 + 28 =
45 + 35 =	56 + 36 =
37 + 44 =	34 + 27 =

Add your own practice items

Students should also be given the opportunity to use base ten blocks to model front-end addition at their tables before being expected to apply this computational strategy mentally.

- **Finding Compatibles (New)**

This strategy for addition involves looking for pairs of numbers that add to ten to make the addition easier. For example, for 3 + 8 + 7, think,"*3 + 7 is 10, and 10 plus 8 is18."*

Practice Items

5 + 4 + 5 =	2 + 4 + 6 + 8 + 3 =
2 + 3 + 8 =	1 + 5 + 6 + 9 + 5 =
4 + 6 + 2 =	7 + 5 + 3 + 4 + 6 =
1 + 9 + 5 =	8 + 4 + 5 + 6 + 5 =
3 + 6 + 7 =	6 + 8 + 9 + 1 + 2 =

Add your own practice items

- **Compensation (New)**

This strategy for addition involves changing one number to the nearest ten, carrying out the addition and then adjusting the answer to compensate for the original change. For example, for 17 + 9, think, "17 plus 10 is 27, but I added one too many; so, I compensate by subtracting 1 to get 26."

Practice Items

2 + 9 =	5 + 8 =	9 + 6 =
3 + 9 =	9 + 5 =	8 + 3 =
9 + 4 =	8 + 7 =	7 + 8 =

Your goal for teaching mental computation should be to show students a wide variety of mental methods, provide opportunities where each method can be employed, and encourage students to use mental methods regularly to improve.

F. Mental Computation – Subtraction

- **Using "Think Addition" in Subtraction (Extension)**

In grade 2 it is appropriate to provide practice items involving the subtraction of 2-digit numbers with only one non-zero digit in each number. For example, for 90 – 30, students should think, "*30 plus what equals 90*?", and use their knowledge of the single-digit addition facts to help determine the answer.

Practice Items

80 – 50 =	90 – 30 =	60 – 30 =
90 – 60 =	60 – 20 =	50 – 20 =
70 – 10 =	40 – 30 =	90 – 10 =
80 – 40 =	70 – 50 =	30 – 20 =

Situations must be regularly provided to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. It is recommended that regular, maybe daily, practice be provided.

Estimation

G. Estimation – Addition and Subtraction

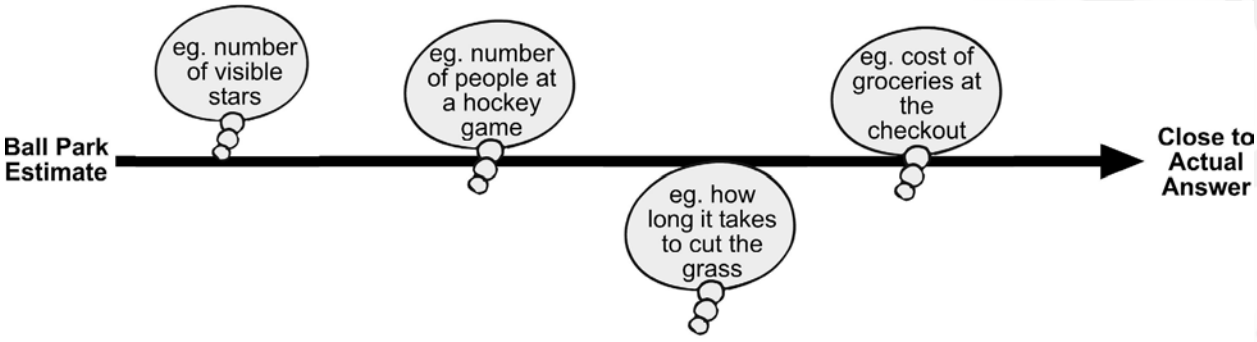
When asked to estimate, students often try to do the exact computation and then "round" their answer to produce an estimate that they think their teacher is looking for. Students need to see that estimation is a valuable and useful skill, one that is used on a daily basis by many people.

Students need to see that estimation is a valuable and useful skill, one that is used on a daily basis by many people.

Estimates can be very broad and general, or they can be quite close to the actual answer. It all depends on the reason for estimating in the first place, and these reasons can vary in context and according to the needs of the individual at the time.

Help students identify situations outside of school where they would estimate distances, number, temperature, length of time and discuss how accurate their estimates needed to be. Place these situations on an estimation continuum with broad, ball-park estimates at one end and estimates that are very close to the actual answer at the other.

For example:



In mathematics, it is essential that estimation strategies are used by students before attempting pencil/paper or calculator computations to help them determine whether or not their answers are reasonable.

When teaching estimation strategies, it is important to use words and phrases such as, *about, almost, between, approximately, a little more than, a little less than, close to* and *near*.

Ongoing practice in computational estimation is a key to developing understanding of numbers and number operations. It is a mental activity; therefore, regular oral practice must be provided.

- **Rounding in Addition and Subtraction (New)**
  This strategy for addition and subtraction involves rounding the highest place value in each number and then adding or subtracting the rounded numbers. To help support short-term memory, it will be necessary for most students to first jot down the rounded numbers and then do the computation mentally.

  At this grade level, numbers which involve 5 or 50 in the rounding procedure are not included in the practice items. This is introduced in grade 4.

  Example
  To estimate 27 + 31, think, “27 rounds to 30 and 31 rounds to 30, so 30 plus 30 is 60. “

  To estimate 87 - 32, think, “87 rounds to 90 and 32 rounds to 30, so 90 subtract 30 is 60.”

  Practice Items

48 + 23 =	34 + 59 =	61 + 48 =
18 + 22 =	97 + 12 =	14 + 32 =
28 + 57 =	41 + 34 =	57 – 14 =
84 – 9 =	82 – 59 =	36 – 22 =
43 – 8 =	54 – 18 =	68 – 34 =
99 – 47 =	93 – 12 =	32 + 59 =

Add your own practice items

Appendixes

Appendix 1

Thinking Strategies in Mental Math

Mental math proficiency represents one important dimension of mathematical knowledge. Not all individuals will develop rapid mental number skills to the same degree. Some will find their strength in mathematics through other avenues, such as visual or graphic representations or creativity in solving problems. But mental math has a clear place in school mathematics. It is an area where many parents and families feel comfortable offering support and assistance to their children.

The following table identifies all of the thinking strategies in *Mental Math: Fact Learning, Mental Computation and Estimation* and the grade level in which they are first introduced. These strategies are then extended and developed in subsequent years.

For example, Front End Addition involving 2-digit numbers is first introduced in grade 2, continued in grade 3, extended to 3-digit numbers in grade 4, and to decimal tenths, hundredths, and thousandths in grades 5 and 6. The teachers guide for each grade level contains a complete description of each strategy with examples and practice items.

Strategy	Description
Grade 1	
Pre-Operation • Patterned Set Recognition • Part-Part-Whole Relationships • Counting On and Back • Next Number • Ten-Frame Visualization for Numbers 0-10 • One More/One Less, Two More/Two Less Relationships	• Students are able to identify common configuration sets of numbers such as the dots on a standard die, dominoes and dot cards without counting. • Recognition of two parts in a whole. Leads to the understanding that numbers can be decomposed into component parts. • Students can count on and back from a given number 0-9. • Students are able to immediately state the number that comes after any given number from 0-9. • Students can visualize the standard ten-frame representation of numbers and answer questions from their visual memories. • Students are presented with a number and asked for the number that is *one more*, *one less*, *two more*, or *two less* than the number.
Addition Facts to 10 • Doubles • Plus 1 Facts • Plus 2 Facts • Plus 3 Facts	• Doubles posters created as visual images • *Next number* facts • Ten-frame, skip counting, 2-more-than relationship, counting on • Ten-frame, 2-more-than plus 1, counting on
Subtraction Facts With Minuends to 10 • Think-Addition • Ten Frame Visualization • Counting Back	• For 9 - 3, think, *"3 plus what equals 9?"* • Visualize the minuend on a ten-frame, remove the subtrahend, to determine the difference. • For -1, -2, -3 facts
Adding 10 to a Number	For numbers 11-20

Grade 2	
Addition Facts to 18 • Near Doubles • 2-Aparts • Plus zero • Make 10	• Double the smaller number and add 1 • Double the number in between • *No change* facts • For facts with 8 or 9 as addends. Eg. 7 + 9 is the same as 10 + 6
Subtraction Facts With Minuends to 18 • Up Through 10 • Back Down Through 10	• For 13 - 8, think, *"From 8 up to 10 is **2**, and then 3 more is **5**."* • For 14 - 6, think, *"14 - 4 gets me to 10, and then 2 more brings me to 8."*
Addition facts extended to numbers in the 10's	2-Apart Facts: 3 + 5 is double 4, so 30 + 50 is double 40.
Front-end Addition	Highest place values are totaled first and then added to the sum of the remaining place values.
Finding Compatibles	Looking for pairs of numbers that add easily, particularly, numbers that add to 10.
Compensation	One or both numbers are changed to make the addition easier and the answer adjusted to compensate for the change.
Rounding in Addition and Subtraction (5 or 50 not involved in rounding process until grade 4)	Round to nearest 10.

Grade 3	
Multiplication Facts With Products to 36 • x 2 facts • Fives • Nifty Nines • Ones • Tricky Zeros • Fours • Threes	Introduced early in 3rd reporting period • Related to the addition doubles • Clock facts, patterns • Patterns, helping fact • No change facts • Groups of zero • Double-double • Double plus 1 more set
Break Up and Bridge	With this front-end strategy, you start with all of the first number and add it to the highest place value in the other number, and then add on the rest.
Front-End Estimation for Addition and Subtraction	Add or subtract just the largest place values in each number to produce a “ball park” estimate.
Adjusted Front-End Estimation for Addition and Subtraction	Same as above, except the other place values are considered for a more accurate estimate.
Grade 4	
Make 10's, 100's, 1000's for addition	48 + 36 is the same as 50 + 34 which is 84
Multiplication Facts With Products to 81 • Last Six Facts	Mastery by year-end For facts not already covered by previous thinking strategies
Subtraction facts extended to numbers in the 10's, 100's 100's	Only 1 non-zero digit in each number eg. 600 - 400 =
Compensation (new for subtraction)	For 17-9, think, *“17 - 10 is 7, but I subtracted 1 too many, so the answer is 8."*
Break Up and Bridge (new for subtraction)	For 92 - 26, think, *“92 - 20 is 72 and then 6 more is 66."*
Multiply by 10 and 100 using a place-value-change strategy	The place values for a number multiplied by 100 *increase* 2 places. Eg. 34 x 100; The 4 ones becomes 4 hundreds and the 3 tens becomes 3 thousand; 3000 + 400 = 3400

Grade 5	
Division Facts With Dividends to 81 • "Think-Multiplication"	Mastery by year-end For 36 ÷ 6, think *"6 times what equals 36?"*
Balancing for a Constant Difference	Involves changing both number in a subtraction sentence by the same amount to make it easier to complete. The difference between the two numbers remains the same. Eg. for 27 - 16, add 3 to each number and think, *"30 - 19 = 11"*
Multiply by 0.1, 0.01, 0.001 using a place-value-change strategy	The place values for a number multiplied by 0.1 *decrease* 1 place. Eg. 34 x 0.1; The 4 ones becomes 4 tenths and the 3 tens becomes 3 ones; 3 and 4 tenths, or 3.4.
Front-End Multiplication (Distributive Principle)	Involves finding the product of the single-digit factor and the digit in the highest place value of the second factor, and adding to this product a second sub-product. 706 x 2 = (700 x 2) + (6 x 2) = 1412
Compensation in Multiplication	Involves changing one factor to a 10 or 100, carrying out the multiplication, and then adjusting the product to compensate for the change. 7 x 198 = 7 x 200 (1400) subtract 14 = 1386
Divide by 10, 100, 1000 using a place-value-change strategy.	The place values for a number divided by 10 *decrease* 1 place. Eg. 34 ÷ 10; The 4 ones becomes 4 tenths and the 3 tens becomes 3 ones; 3 and 4 tenths, or 3.4.
Rounding in Multiplication	Highest place values of factors are rounded and multiplied. When both numbers are close to 5 or 50, one number rounds up and the other down.

Grade 6	
Divide by 0.1, 0.01, 0.001 using a place-value-change strategy	The place values for a number divided by 0.01 *increase* 2 places. Eg. 34 ÷ 0.01; The 4 ones becomes 4 hundreds and the 3 tens becomes 3 thousand; 3000 + 400 = 3400
Finding Compatible Factors (Associative Property)	Involves looking for pairs of factors, whose product is easy to work with, usually multiples of 10. For example, for 2 x 75 x 500, think, “2 x 500 = 1000 and 1000 x 75 is 75 000.
Halving and Doubling	One factor is halved and the other is doubled to make the multiplication easier. Students would need to record sub-steps. For example, 500 x 88 = 1000 x 44 = 44 000.
Using division facts for 10's, 100's 1000's	Dividends in the 10's, 100's, and 1000's are divided by single digit divisors. The quotients would have only one digit that wasn’t a zero. For example, for 12 000 ÷ 4, think single digit division facts. 12 ÷ 4 = 3, and thousands divided by ones is thousands, so the answer is 3000.
Partitioning the Dividend (Distributive Property)	The dividend is broken up into two parts that are more easily divided by the divisor. For example, for 372 ÷ 6, think, “(360 + 12) ÷ 6, so 60 + 2 is 62.”

Appendix 2
Mental Math: Fact Learning, Mental Computation, Estimation (Scope and Sequence)

	GRADE 1	GRADE 2	GRADE 3	GRADE 4	GRADE 5	GRADE 6
FACT LEARNING	**Pre-Operation Strategies:** ‣ Patterned Set Recognition for numbers 1-6 (not dependent on counting) ‣ Part-Part-Whole Relationships ‣ Counting on, Counting Back ‣ Next Number ‣ Ten Frame Recognition and Visualization for Numbers 0-10 ‣ One More/One Less and Two More/Two Less Relationships **Addition Facts With Sums to 10 Thinking Strategies:** ‣ Doubles ‣ Plus 1 Facts ‣ Plus 2 Facts ‣ Plus 3 Facts ‣ Ten Frame Facts **Subtraction Facts With Minuends to 10 Thinking Strategies** ‣ Think-Addition ‣ Ten Frame Facts ‣ Counting Back	**Addition and Subtraction Facts** ‣ mastery of facts with sums and minuends to 10 by mid-year ‣ mastery of facts with sums and minuends to 18 by year end **New Thinking Strategies for Addition** ‣ Near Doubles ‣ 2-Apart Facts ‣ Plus 0 Facts ‣ Make 10 Facts **New Thinking Strategies for Subtraction Facts** ‣ Up Through 10 ‣ Back Down Through 10	**Addition** ‣ Review and reinforce facts with sums to 18 and thinking strategies ‣ Addition facts extended to 2-digit numbers. Think *single-digit addition facts* and apply the appropriate place value. **Subtraction** ‣ Review and reinforce facts with minuends to 18 and thinking strategies. ‣ Subtraction facts extended to 2-digit numbers. Think *single-digit subtraction facts* and apply the appropriate place value. **Multiplication Facts (**Products to 36) **Thinking Strategies:** ‣ x2 Facts (related to addition doubles) ‣ x10 Facts (patterns) ‣ x5 Facts (clock facts, patterns) ‣ x9 Facts (patterns, helping facts) ‣ x1 Facts ("no-change" facts) ‣ x0 Facts (products of zero) ‣ x4 Facts (double-double) ‣ x3 Facts (double plus 1 set)	**Addition** Review and reinforce facts to 18 and thinking strategies **Subtraction** ‣ Review and reinforce facts with minuends to 18 and thinking strategies **Multiplication** ‣ Facts With Products to 36-Mastery by Mid-Year ‣ Facts With Products to 81-Mastery by Year End **Thinking Strategies:** ‣ x2 Facts (related to addition doubles) ‣ x10 Facts (patterns) ‣ x5 Facts (clock facts, patterns) ‣ x9 Facts (patterns, helping facts) ‣ x1 Facts ("no-change" facts) ‣ x0 Facts (products of zero) ‣ x4 Facts (double-double) ‣ x3 Facts (double plus 1 set) ‣ Last Six Facts (New; various strategies)	Review Addition and Subtraction Facts With Sums/Minuends to 18 **Multiplication** Review and Reinforce Multiplication Facts With Products to 81 and Thinking Strategies **Division** Division Facts With Dividends to 81-Mastery by Year End Using a "Think-Multiplication" Strategy	‣ Review Addition Subtraction, Multiplication and Division Facts. ‣ Reintroduce thinking strategies to struggling students ‣ See the Mental Math Teacher's Guides for Grades 2-5 for strategies and practice items
MENTAL COMPUTATION	**Addition:** ‣ Adding 10 to a number without counting	**Addition** ‣ Addition facts extended to 2-digit numbers. Think *single-digit addition facts* and apply the appropriate place value. (New) ‣ Front End Addition (2-digit numbers) ‣ Finding Compatibles (single-digit number combinations that make 10) ‣ Compensation (single-digit numbers) **Subtraction** ‣ *Think-Addition* (extended to 2-digit numbers)	**Addition** ‣ Front End Addition (continued from Grade 2) ‣ Break Up and Bridge (New) ‣ Finding Compatibles (single digit numbers that add to 10; 2-digit numbers that add up to 100) ‣ Compensation (extended to 2-digit numbers) **Subtraction** ‣ Back Down Through 10s (extended to subtraction of a single digit from a 2-digit number) ‣ Up Through 10s (extended to 2-digit numbers)	**Addition** ‣ Facts Extended to Addition of Numbers in 10s, 100s, and 1000s ‣ Front End Addition (extended to numbers in 1000s) ‣ Break Up and Bridge (extended to numbers in 100s ‣ Finding Compatibles (extended to numbers in 1000s) ‣ Compensation (extended to numbers in 100s) ‣ Make 10s, 100s, 1000s (Extension) **Subtraction** ‣ Facts Extended to Subtraction of Numbers in 10s, 100s, 1000s ‣ Back Down Through 10s (extended to numbers in 100s) ‣ Up Through 10s (extended to numbers in the 100s) ‣ Compensation (New for Subtraction) ‣ Break Up and Bridge (New for Subtraction) **Multiplication** ‣ Multiplying by 10 and 100 using a "place-value-change" strategy rather than an "attach zeros" strategy	**Addition** ‣ Front End Addition (extended to decimal 10^{ths} and 100^{ths}) ‣ Break Up and Bridge (extended to numbers in 1000s and to decimal 10^{ths} and 100^{ths}) ‣ Finding Compatible (extended to decimal 10^{ths} and 100^{ths}) ‣ Compensation (extended to 1000s and to decimal 10^{ths} and 100^{ths}) ‣ Make 10s, 100s, 1000s (continued from Grade 4) **Subtraction** ‣ Back Down Through 10s, 100s, 1000s (Extension) ‣ Up Through 10s - (extended to Numbers in 1000s and to decimal 10^{ths} and 100^{ths}) ‣ Compensation - (extended to numbers in 1000s) ‣ Balancing for a constant difference (New) ‣ Break Up and Bridge (extended to numbers in 1000s) **Multiplication** ‣ Facts Extended to 10s, 100s and 1000s ‣ Multiplying by 10, 100, 1000 using a "Place-Value-Change" strategy, rather than an "attach zeros" strategy - (continued from Grade 4) ‣ Multiplying by 0.1, 0.01 and 0.001 using a place-value-change strategy (New) ‣ Front End Multiplication (New) ‣ Compensation (New for Multiplication)	**Addition** Practice items provided for review of mental computation strategies for addition. ‣ Front End ‣ Break Up and Bridge ‣ Finding Compatibles ‣ Compensation ‣ Make 10s, 100s, 1000s **Subtraction** ‣ Back Down Through 10s, 100s, 1000s ‣ Up Through 10s, 100s, 1000s ‣ Compensation ‣ Balancing for a Constant Difference (continued From Grade 5) ‣ Break Up and Bridge (extended to numbers in the 10 000s) **Multiplication and Division** ‣ Multiplying and Dividing by 10, 100, 1000 using a "place-value-change" strategy) ‣ Multiplying by 0.1, 0.01, 0.001 (continued from Grade 5) ‣ Dividing by 0.1, 0.01, 0.001 using a "place-value-change" strategy (New) ‣ Front End Multiplication (continued from Grade 5) ‣ Compensation (continued from Grade 5) ‣ Finding Compatible Factors (New) ‣ Halving and Doubling (New) ‣ Using Division Facts for 10s, 100s, 1000s (New) Dividends of 10s, 100s, 1000s divided by single-digit divisors. ‣ Partitioning The Dividend (New)
ESTIMATION		‣ Rounding in Addition and Subtraction (2-digit numbers; 5 is not involved in the rounding procedure until Grade 4)	‣ Front End Addition and Subtraction (New) ‣ Rounding in Addition and Subtraction (extended to 3-digit numbers; 5 or 50 not involved in the rounding procedure until Grade 4) ‣ Adjusted Front End in Addition and Subtraction (New)	‣ Rounding in Addition and Subtraction (extended to 4-Digit Numbers and involving 5, 50 and 500 in the rounding procedure) ‣ Adjusted Front End in Addition and Subtraction (extended to numbers in 1000s)	‣ Rounding in Addition and Subtraction (continued from Grade 4 ‣ Rounding in Multiplication (2-or-3- digit factor by single digit factor; 2-digit by 2-digit) ‣ Adjusted Front End for Addition and Subtraction (extended to decimal 10^{ths} and 100^{ths})	‣ Rounding in Addition and Subtraction (continued From Grade 5) ‣ Rounding in Multiplication (extended from Grade 5 to include 3-digits by 2-digits) ‣ Rounding in Division (New)

Printed by the Document Publishing Centre
Design: Strategic Marketing and Graphic Design
2008

08ED41-19785